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MATHEMATICS
SPECIALIST

UNITS
1&2



Mathematics Specialist Units 1 and 2
1st Edition
A. J. Sadler

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Preface.

This text targets units one and two of the West Australian course *Mathematics Specialist*. Chapters one to eight cover the content of Unit One and chapters nine to thirteen cover Unit Two.

UNIT ONE	UNIT TWO	UNIT THREE	UNIT FOUR
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The West Australian course, *Mathematics Specialist*, is based on the Australian National Curriculum Senior Secondary course *Specialist Mathematics*. At the time of writing there are only small differences between the content for units one and two of the West Australian course and units one and two of the Australian Curriculum course, so this text would also be suitable for anyone following units one and two of the Australian Curriculum course, *Specialist Mathematics*. The main differences being that the West Australian units include vertical translations of trigonometric graphs, solving linear equations in two variables by matrix methods, and when considering converse and contrapositive statements the inverse is also covered.

The book contains text, examples and exercises containing many carefully graded questions. A student who studies the appropriate text and relevant examples should make good progress with the exercise that follows.

The book commences with a section entitled *Preliminary work*. This section briefly outlines work of particular relevance to this unit that students should either already have some familiarity with from the mathematics studied in earlier years, or for which the brief outline included in the section may be sufficient to bring the understanding of the concept up to the necessary level.

As students progress through the book they will encounter questions involving this preliminary work in the *Miscellaneous Exercises* that feature at the end of each chapter. These miscellaneous exercises also include questions involving work from preceding chapters to encourage the continual revision needed throughout the unit.

Some chapters commence with a "Situation" or two for students to consider, either individually or as a group. In this way students are encouraged to think and discuss a situation, which they are able to tackle using their existing knowledge, but which acts as a fore-runner and stimulus for the ideas that follow. Students should be encouraged to discuss their solutions and answers to these situations and perhaps to present their method of solution to others. For this reason answers to these situations are generally not included in the book.

At times in this series of texts I have found it appropriate to perhaps go a little outside the confines of the syllabus for the unit involved. In this regard readers will find that in unit two I consider $\sin P \pm \sin Q$ and $\cos P \pm \cos Q$, and when considering matrix equations of the form $AX = B$, my considerations go beyond X and B being column matrices. I also include shear transformations and mention proof by exhaustion.

Alan Sadler

Contents

Unit One.

Preliminary work.	Page 7
Number.	7
The absolute value.	7
Trigonometry.	8
Use of algebra.	8
Similar triangles.	9
Congruent triangles.	11
Sets and Venn diagrams.	12
1. True or false?	Page 13
True or false?	14
Converse, inverse and contrapositive.	18
Miscellaneous Exercise One.	21
2. Counting.	Page 23
Permutations (arrangements).	23
Factorial notation.	24
Permutations of objects from a group of objects, all different.	25
Permutations of objects, not all different.	29
Addition principle.	30
Inclusion – exclusion principle.	31
Arrangements of objects with some restriction imposed.	35
I. Multiplicative reasoning.	35
II. Addition and multiplicative reasoning.	40
Combinations.	45
${}^n C_r$ and Pascal's triangle.	54
Miscellaneous Exercise Two.	55
3. Vectors – basic ideas.	Page 57
Vector quantities.	58
Adding vectors.	62
Mathematical representation of a vector quantity.	66
Equal vectors.	67
The negative of a vector.	67
Multiplication of a vector by a scalar.	68
Parallel vectors.	68
Addition of vectors.	68
Subtraction of one vector from another.	69
The zero vector.	69
$ha = kb$.	70
Miscellaneous Exercise Three.	76

4. Vectors in component form.	Page 77
Further examples.	86
Position vectors.	92
Miscellaneous Exercise Four.	96
5. Geometric proofs.	Page 97
Proof.	97
Definitions, axioms and theorems.	97
Circle properties.	99
Angles in circles.	100
Tangents and secants.	106
Miscellaneous Exercise Five.	111
6. Relative displacement and relative velocity.	Page 113
Relative displacement.	115
Relative velocity.	118
Miscellaneous Exercise Six.	124
7. Proofs using vectors.	Page 127
Miscellaneous Exercise Seven.	131
8. Scalar product.	Page 133
Scalar product.	133
Algebraic properties of the scalar product.	134
The scalar product from the components.	138
Proofs using the scalar product.	143
Miscellaneous Exercise Eight.	146

Unit Two.

Preliminary work.	Page 150
Radian measure.	150
Unit circle definitions of $y = \sin x$, $y = \cos x$ and $y = \tan x$.	150
Transformations of $y = \sin x$ (and of $y = \cos x$ and $y = \tan x$).	154
Angle sum and angle difference identities.	155
Solving trigonometric equations.	158
9. Trigonometrical identities and equations.	Page 159
The double angle identities.	164
$a \cos \theta + b \sin \theta$.	167
$\sec \theta$, $\operatorname{cosec} \theta$ and $\cot \theta$.	170
Product to sum and sum to product.	174
General solutions of trigonometric equations.	177
Obtaining the rule from the graph.	181
Modelling periodic motion.	182
Miscellaneous Exercise Nine.	186

10. Matrices.	Page 187
Adding and subtracting matrices.	189
Multiplying a matrix by a number.	189
Equal matrices.	190
Multiplying matrices.	193
Zero matrices.	201
Multiplicative identity matrices.	203
The multiplicative inverse of a square matrix.	204
Using the inverse matrix to solve systems of equations.	210
Miscellaneous Exercise Ten.	214
11. Transformation matrices.	Page 217
Transformations and matrices.	217
Determining the matrix for a particular transformation.	220
The determinant of a transformation matrix.	220
The inverse of a transformation matrix.	220
Combining transformations.	222
Further examples.	223
A general rotation about the origin.	227
A general reflection in a line that passes through the origin.	227
Miscellaneous Exercise Eleven.	229
12. Proof.	Page 231
Proof by exhaustion.	234
Proof by induction.	236
Extension activity: Investigating some conjectures.	241
Miscellaneous Exercise Twelve.	242
13. Complex numbers.	Page 245
Complex numbers.	247
Complex number arithmetic.	250
The conjugate of a complex number.	251
Equal complex numbers.	251
Linear factors of quadratic polynomials.	252
Argand diagrams.	255
Miscellaneous Exercise Thirteen.	257
Answers	261
Index	284

Important note.

This text has been written based on my interpretation of the appropriate Mathematics Specialist syllabus documents as they stand at the time of writing. It is likely that as time progresses some points of interpretation will become clarified and perhaps even some changes could be made to the original syllabus. I urge teachers of the Specialist Mathematics course, and students following the course, to check with the appropriate curriculum authority to make themselves aware of the latest version of the syllabus current at the time they are studying the course.

Alan Sadler